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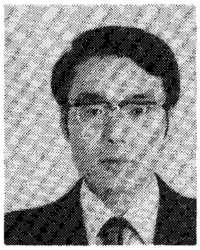

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Prediction of Laser Wavelength for Minimum Total Dispersion in Single-Mode Step-Index Fibers

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Abstract —**Pulse dispersion in single-mode optical fibers with step-index profiles has been analyzed in the past using asymptotic methods. One of these methods is based on the approximate characteristic equation for the dominant mode of propagation in these structures, obtained using the "weakly guided" condition. Other methods use approximations for certain parameters of this equation. Utilizing numerical methods of differentiation and interpolation, we have developed a method for the analysis of pulse dispersion in these fibers that is based on solutions of the exact characteristic equation. Exact formulas for the parameters necessary for this study have been established and developed to the point where the steps that would follow, involving extensive analytical effort, are replaced by computational procedures. We make comparisons between our method and those that, although based on asymptotic expressions, present the best theoretical characteristics. The differences found are discussed. This method permits greater precision in prediction of the ideal laser wavelength for use with a given single-mode optical fiber.**

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I. INTRODUCTION

DISTORTION OF pulses in single-mode optical fibers with step-index profiles results from a combination of dispersive effects that are due to the wavelength (λ) dependence of the refractive indexes of the lightguide

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materials and also due to the wavelength dependence of the group delay of the single propagating mode.

The first effect is known as material dispersion and depends only on the materials used in the fiber. In this paper we assume that the refractive indexes of both core and cladding, n_1 and n_2, respectively, follow the three-term Sellmeier equation [1]

$$n_j^2 = 1 + \sum_{i=1}^{3} \frac{A_i \lambda^2}{\lambda^2 - l_i^2}, \qquad j = 1,2 \tag{1}$$

where A_i are constants related to the number of particles in the material that can oscillate at wavelengths l_i.

The second effect, called waveguide dispersion, is a consequence only of the waveguiding properties of the optical fiber. This dispersion depends on the core radius, the propagation constant of the selected dominant mode (HE_{11}), and some of its derivatives. By definition [2], this waveguide dispersion is computed keeping the refractive indexes of the core and cladding constant with wavelength.

The combination of the above two dispersive effects gives the total dispersion (or, simply, dispersion) of pulses bounded in a single-mode fiber. Since this dispersion limits the useful bandwidth of optical communication systems, one should use a wavelength that minimizes this effect.

The wavelength for minimum dispersion $\hat{\lambda}$ has been computed using asymptotic formulas. This approach has been used because, in most cases, the relative difference of refractive indexes of the core and cladding, given by

$$\Delta = (n_1 - n_2)/n_2 \tag{2}$$

is small.

The condition $\Delta \ll 1$ allows a substantial simplification of the exact characteristic equation that is the starting point for the dispersion analysis of the selected optical fiber. Two asymptotic procedures have been employed which we will briefly describe as follows:

1) One involves working with the values of the dominant mode propagation constant obtained directly from the solution of the approximate characteristic equation and using first analytical expressions and then numerical methods to obtain the values of some of the derivatives of the dominant mode propagation constant.

2) The other involves working with approximations for the parameters of the approximate characteristic equation (these are analytically simple formulas) and using these expressions to obtain the values of the HE_{11} mode propagation constant and some of its derivatives.

These procedures lead to some results which are not very satisfying [3].

In this paper we describe an analytical and computational procedure to obtain $\hat{\lambda}$. Exact expressions for the necessary parameters involved are developed up to the point where subsequent deductions would be very laborious; at this stage, the analytical effort was replaced by numerical differentiation and interpolation. We also compare our results with some approximate methods that, in our opinion [4], show the best theoretical characteristics. Two of these methods [2], [5], [6] use Procedure (1) above, while the other [7] uses Procedure (2).

In Section II, we present the analytical formalism used in our analysis and, in Section III, give an outline of the computational procedure. In Section IV we make some numerical comparisons between our method and some methods based on approximate formulas.

II. Exact Equations for Total Dispersion Analysis

Total dispersion, as discussed in Section I, is given by [4]

$$D_T = \frac{1}{c}\frac{dN_T}{d\lambda} \tag{3}$$

where c is the speed of light in free space and N_T is the total group index given by

$$N_T = \frac{1}{n_e}\left\{ n_2 N_2 + \left(\frac{V}{2}\frac{db}{dV} + b \right)\theta \right\} \tag{4}$$

where

$$\theta = n_1 N_1 - n_2 N_2 \tag{5}$$

$$N_i = n_i - \lambda \frac{dn_i}{d\lambda}, \qquad i = 1,2 \tag{6}$$

$$n_e = \left\{ n_2^2 + (n_1^2 - n_2^2) b \right\}^{1/2}. \tag{7}$$

In (4)–(7), b is the normalized propagation constant for the HE_{11} mode, V is a normalized frequency, $N_i (i = 1,2)$ are the group indexes of the core and cladding, respectively, and n_e is the effective phase index, i.e., the phase index "seen" by the HE_{11} mode propagating in the optical fiber under consideration.

The normalized propagation constant is given by

$$b = W^2/V^2 = 1 - U^2/V^2 \tag{8}$$

where

$$V = \frac{2\pi a}{\lambda}(n_1^2 - n_2^2)^{1/2} \tag{9}$$

and a is the core radius of fiber. The parameter U (or W) comes from the solution of the exact characteristic equation [8]

$$(J^+ + K^+)(\epsilon J^- - K^-) + (J^- - K^-)(\epsilon J^+ + K^+) = 0 \tag{10}$$

where

$$J^+ = \frac{J_{\nu+1}(U)}{U J_\nu(U)} \qquad J^- = \frac{J_{\nu-1}(U)}{U J_\nu(U)} \tag{11}$$

$$K^+ = \frac{K_{\nu+1}(W)}{W K_\nu(W)} \qquad K^- = \frac{K_{\nu-1}(W)}{W K_\nu(W)} \tag{12}$$

using ν equal to unity. In (11) and (12), J and K are Bessel functions and modified Hankel functions, respectively. The electrical permittivity in (10) is given by

$$\epsilon = n_1^2 / n_2^2. \tag{13}$$

Taking into account the wavelength dependence of the parameters involved in the total group index, (3) becomes

$$D_T = \frac{1}{c}\left\{\frac{A_1A_2 - A_3A_4}{A_5}\right\} \quad (14)$$

where

$$A_1 = n_e \quad (15)$$

$$A_2 = \phi_2 + (\phi_1 - \phi_2)\left(\frac{V}{2}\frac{db}{dV} + b\right) - \frac{V\theta^2}{2\lambda(n_1^2 - n_2^2)}\left\{V\frac{d^2b}{dV^2} + 3\frac{db}{dV}\right\} \quad (16)$$

$$A_3 = n_2N_2 + \left(\frac{V}{2}\frac{db}{dV} + b\right)\theta \quad (17)$$

$$A_4 = \frac{1}{n_e}\left\{(1-b)n_2n_2' - \frac{V\theta}{2\lambda}\frac{db}{dV} + n_1n_1'b\right\} \quad (18)$$

$$A_5 = n_e^2 \quad (19)$$

and

$$\phi_j = N_jn_j' - \lambda n_jn_j'' \quad (20a)$$

$$n_j' = -\frac{1}{n_j}\sum_{i=1}^{3}\frac{A_il_i^2\lambda}{(\lambda^2 - l_i^2)^2} \quad (20b)$$

$$n_j'' = \frac{1}{n_j}\left\{-(n_j')^2 + \sum_{i=1}^{3}\frac{A_il_i^2(3\lambda^2 + l_i^2)}{(\lambda^2 - l_i^2)^3}\right\}. \quad (20c)$$

The prime on n_j indicates differentiation with respect to wavelength. The subscript j can be either 1 or 2.

To operate the fiber at the maximum transmission rate, we must select the wavelength that corresponds to minimum total dispersion. Thus we select λ such that

$$D_T|_{\lambda=\hat{\lambda}} = 0. \quad (21)$$

In Section III we show the computational procedure used to solve (21).

III. Computational Procedure

We implemented a computer program (Fortran-IV language, double precision) to solve (21) for $\hat{\lambda}$. Our program accepts as inputs either of two sets of parameters: a) core radius a, in micrometers; relative difference between refractive indexes Δ given by (2); coefficients (A_i, l_i) of Sellmeier's three-term equation for the fiber cladding; or b) core radius a, in micrometers; coefficients of Sellmeier's three-term equation for the core $(A_i, l_i - N)$ and for the cladding $(A_i, l_i - C)$.

In case a), by knowing Δ, it is possible to compute the core phase index using

$$n_1 = (1 + \Delta)n_2. \quad (22)$$

In case b), the phase indexes of the core and cladding are fixed *a priori* by Sellmeier's coefficients for both materials. Note that while Δ is fixed in case a), this parameter is wavelength dependent in case b). To consider Δ constant with wavelength is a theoretical abstraction, and in our computer program we only considered this case for comparison with Chang's results [5], [6].

Values of the normalized propagation constant for the dominant mode, (8), were computed for wavelengths in the range

$$0.8 \leq \lambda \leq 2.0 \quad \mu m \quad (23)$$

using the solutions of the exact characteristic equation. For this purpose we used some standard Scientific Subroutine Package (SSP) subroutines [9] such as BESJ and BESK, modified for double precision, for the computation of the functions J and K, respectively, and DRTMI (Mueller's method) to solve the transcendental equation (10).

Due to analytical complexity, the values of db/dV for each normalized frequency (a vector) were computed by subroutine DDGT3 of SSP; a second entry in this same subroutine gives d^2b/dV^2. These two derivatives are used in the computation of the total dispersion D_T. Knowing the vectors D_T and λ, subroutine DRTMI is used again, supplemented by another that uses Lagrange interpolation to compute the value of $\hat{\lambda}$. All this computation is done in approximately one minute on a PDP-10 computer in the time-sharing mode.

Other computer programs needed for comparison purposes were implemented based on the works of Chang [5], [6], Marcuse [2], and South [7], using the same approach selected by these authors for the analysis of total dispersion. These results are compared with ours in Section IV.

IV. Numerical Results

In this section, the results found using the procedures of Sections II and III are compared with those using asymptotic expressions [2], [6], [7] that can be shown to have the best theoretical characteristics [4]. The coefficients of the Sellmeier equation for the materials used for comparison are shown in Table I [10]–[12].

First, we will compare our results with those derived from Chang [6] (Figs. 1–4). This author uses the "weakly guiding" formula for the characteristic equation to obtain, using (8), the value of b. Its derivatives of b with respect to V are found through complex analytical deductions. He also gives a simplified expression for the total group index that results in an approximate formula for D_T.

Table II shows the values of $\hat{\lambda}$ and V obtained using our method together with the input parameters for option a) of Section III. Fig. 1 shows the variation of $\hat{\lambda}$ with Δ. In this figure the core radius was set equal to 5.3 μm. Note that there are two ranges of Δ where our result (curve labeled *EXACT*) deviates somewhat from Chang's (curve labeled *CHANG*) [6]. For Δ in the range 0.4–0.5 percent the relative errors are approximately 0.06 percent. For values of Δ larger than 0.9 percent the two curves tend to separate. In particular, for the limit shown in Fig. 1, $\Delta = 0.96$ percent and the relative error becomes -0.13 percent ($\hat{\lambda} = 1.3287$ μm, our method ; $\hat{\lambda} = 1.3304$ μm, Chang's method).

Fig. 2 shows the variation of $\hat{\lambda}$ with the core radius a. In

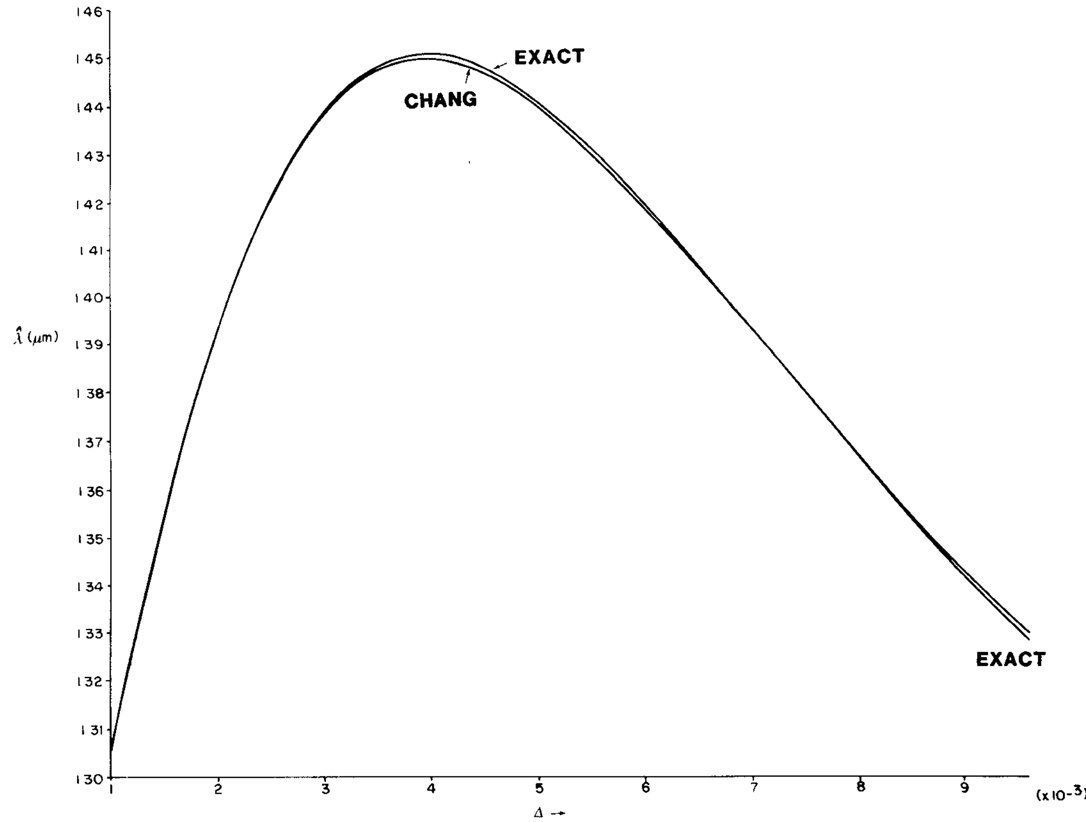

Fig 1. Variation of the wavelength for minimum total dispersion $\hat{\lambda}$ as a function of the relative difference, Δ. The fiber has a core diameter of $2a = 5.3$ μm and Sellmeier coefficients as in Sample 2, Table I, as in [6].

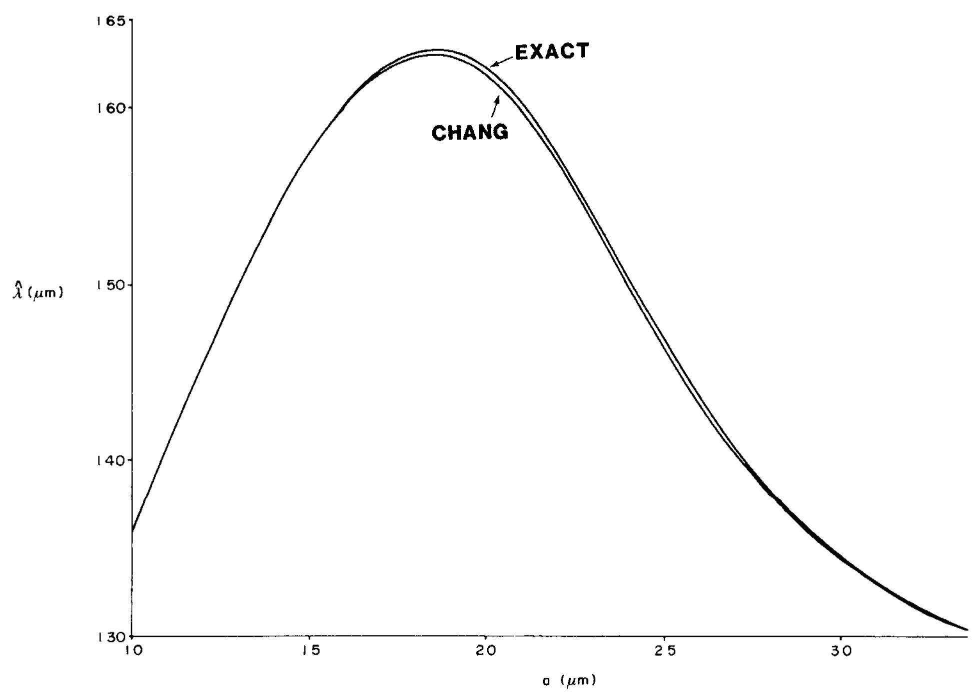

Fig. 2. Variation of the wavelength for total minimum dispersion $\hat{\lambda}$ as a function of the core radius a. This calculation is also based on Sample 2, Table I, using a value of $\Delta = 0.6$ percent.

TABLE I
COEFFICIENTS OF THREE-TERM SELLMEIER EQUATION (SAMPLE 1: ANNEALED; SAMPLE 2: QUENCHED SAMPLE)

SAMPLE	COMPOSITION			COEFFICIENTS OF THE THREE-TERM SELLMEIER EQUATION					
	GeO_2	B_2O_3	SiO_2	A_1	ℓ_1	A_2	ℓ_2	A_3	ℓ_3
1	-	-	100	0.6961663	0.0684043	0.4079426	0.1162414	0.8974794	9.896161
2	-	-	100	0.69675	0.069066	0.408218	0.115662	0.890815	9.900559
3	13.5	-	86.5	0.73454395	0.08697693	0.42710828	0.11195191	0.82103399	10.84654
4	7.0	-	93.0	0.6869829	0.078087582	0.44479505	0.1155184	0.79073512	10.436628
5	-	13.3	86.7	0.690618	0.0619	0.401996	0.123662	0.898817	9.09896

TABLE II
VALUES OF $\hat{\lambda}$ AND V OBTAINED USING OUR METHOD (FIBERS FROM [5] AND [6])

SAMPLE	CORE DIAMETER (μm)	RELATIVE DIFFERENCE Δ	TOTAL DISPERSION $\hat{\lambda}$ (μm)	TOTAL DISPERSION V
1	5.3	0.006	1.4144	1.8669
1	5.3	0.00484	1.4386	1.6478
1	10.0	0.00104	1.3112	1.5813
2	5.3	0.006	1.4201	1.8599
2	5.3	0.00484	1.442	1.6418
2	10.0	0.00104	1.3147	1.5775
2	9.4	0.0019	1.3094	2.0128
5	5.3	0.006	1.3136	2.0049
5	5.3	0.00242	1.3358	1.2507
5	10.0	0.00055	1.2462	1.2064

TABLE III
VALUES OF $\hat{\lambda}$ AND V FOR SAMPLE 2 USING $\Delta = 2.15$ PERCENT (FIRST LINE: OUR APPROACH; SECOND LINE: METHOD OF [6])

CORE DIAMETER 2a (μm)	RELATIVE DIFFERENCE Δ	$\hat{\lambda}$ (μm)	V
3.63	0.0215	1.4691	2.3389
3.63	0.0215	1.4877	2.2972

this case we used $\Delta = 0.6$ percent. We observe a range of core radii where a small difference occurs between our results and Chang's results. For the scale of Fig. 2 the range of a values where this difference is noticeable extends from 1.75–2.70 μm, approximately. For example, the relative error in $\hat{\lambda}$ for $a = 2.0$ μm is about 0.12 percent while for $a = 2.7$ μm this error drops to approximately 0.04 percent.

Using Fig. 1 or 2 it is possible to design a single-mode optical fiber with a step-index profile to give a minimum dispersion for the wavelength of the available optical source using one of two approaches: 1) vary Δ for a fixed core radius; or 2) vary the core radius for a fixed Δ. In addition, we can see from the above figures that only a small error will be made by using Chang's approach in the range analyzed.

Based on these considerations, Chang [5] designed a single-mode fiber with a step-index profile to obtain a minimum dispersion for $\hat{\lambda} = 1.55$ μm. This value, 1.55 μm, is the wavelength for which the lowest attenuation was obtained for this type of structure [13]. Instead of using the materials to get this lowest attenuation [13], Chang designed his fiber using Sample 2 of Table I. In [5], Chang gives the values of core radius and Δ to obtain minimum dispersion at 1.55 μm.

To check the results shown in [5], we used our program

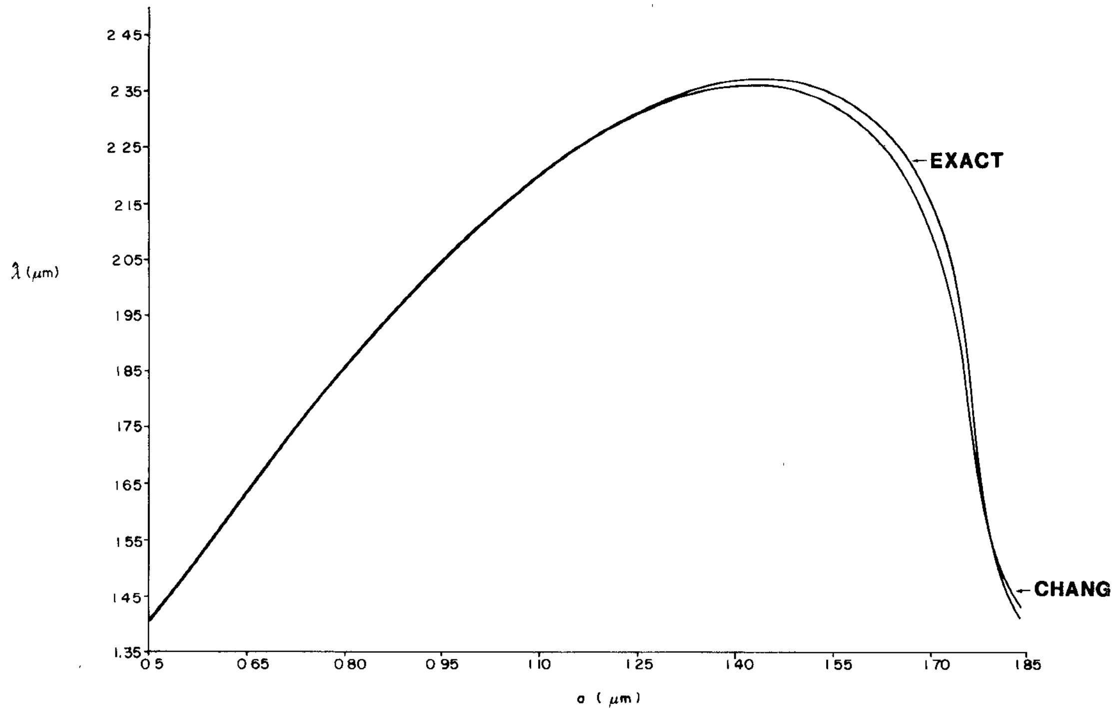

Fig. 3. Variation of the wavelength for total minimum dispersion $\hat{\lambda}$ as a function of core radius a for $\Delta = 2.15$ percent and Sellmeier coefficients of Sample 2.

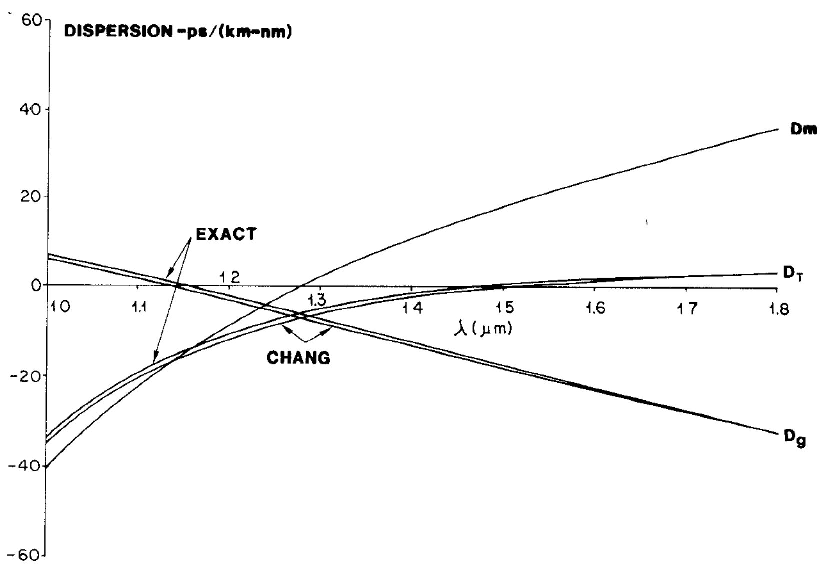

Fig. 4. Dispersion curves for a fiber constructed using Sample 2 material. D_m is the material dispersion, D_g is the waveguide dispersion, and D_T is the total dispersion. These calculations were made using a core diameter of $2a = 3.63$ μm and a relative difference $\Delta = 2.15$ percent.

(exact method) and a computer program that we have written following the instructions given in [6]. The two results are shown in Table III. We noted that, besides the obvious difference in Table III, there is also a noticeable difference between what Chang shows [5] and the corresponding value of Table III (following Chang's approach). Since the results of Table II (our approach) and those shown in [6] are practically identical, we conclude that there is an error in [5].

Fig. 3 shows the variation of $\hat{\lambda}$ with the core radius a for $\Delta = 2.15$ percent. For $a = 1.70$ μm, for instance, the relative error in $\hat{\lambda}$ between our approach and Chang's approach is approximately 2.2 percent while for $a = 1.75$ μm this error increases to about 3.7 percent.

Fig. 4 shows the variation of material (D_m), waveguide (D_g), and total (D_T) dispersion for a fiber with $\Delta = 2.15$ percent and $2a = 3.63$ μm according to our approach and to Chang's approach. The exact analytical expressions for

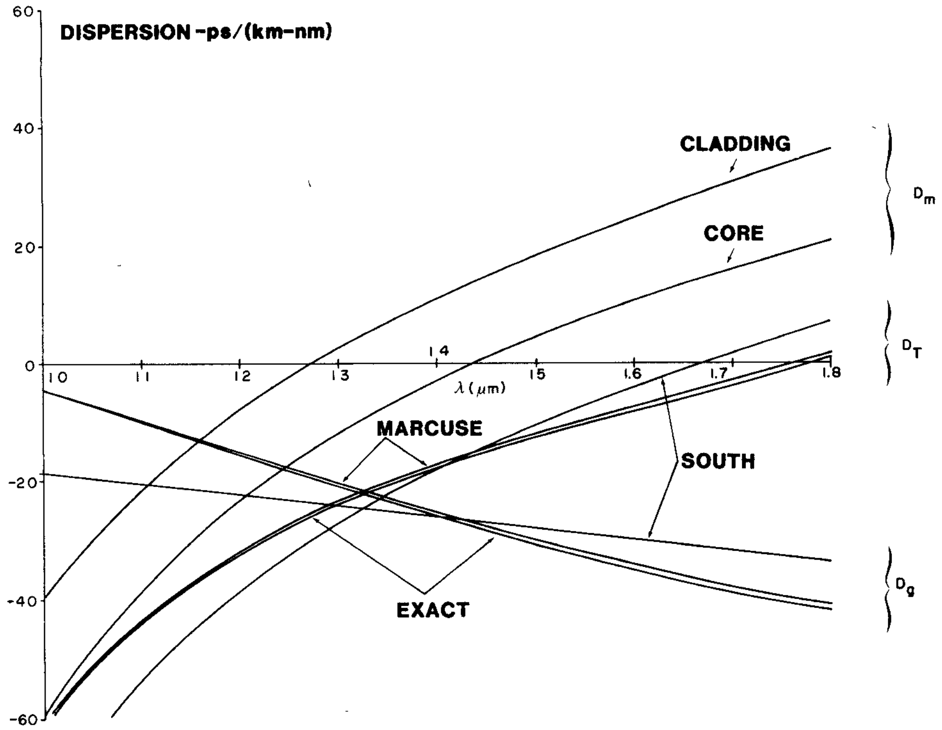

Fig. 5. Dispersion curves for a fiber constructed from 13.5 percent GeO_2–86.5 percent SiO_2 (core, Sample 3, Table I) and 100 percent SiO_2 (cladding, Sample 1, Table I). D_m, D_g, and D_T are as previously defined. The core radius is $a = 1.75$ μm.

TABLE IV
VALUES OF $\hat{\lambda}$ AND V FOR FIBERS USING ANNEALED SiO_2 CLADDING (E = OUR METHOD, M = MARCUSE, S = SOUTH)

CORE MATERIAL	CORE RADIUS (μm)	$\hat{\lambda}$ (μm)	V	METHOD
13.5% GeO_2 86.5% SiO_2	1.75	1.7689	1.5898	E
		1.7542	1.6017	M
		1.6661	1.6781	S
13.5% GeO_2 86.5% SiO_2	2.00	1.5646	2.0316	E
		1.5576	2.0399	M
		1.5878	2.0042	S
7.0% GeO_2 93.0% SiO_2	2.00	1.5562	1.4671	E
		1.5533	1.4694	M
		1.5026	1.5126	S
7.0% GeO_2 93.0% SiO_2	2.25	1.4923	1.7120	E
		1.4901	1.7142	M
		1.4662	1.7389	S

D_m and D_g are shown in the Appendix.

We also used the approaches of Marcuse [2] and South [7] for comparison with ours. The approximate methods of these authors consider $d\Delta/d\lambda \neq 0$, which is a realistic assumption, together with the exact formula for total dispersion. Marcuse [2] uses the "weakly-guiding" expression for the characteristic equation to compute the parameter U (equal to κa in [2]) and a numerical method using a polynomial approximation to compute the derivatives of this variable. We introduced a small modification [4] in Marcuse's computational procedure using analytical expressions for the derivatives of U following some theoretical considerations presented by Snyder [14]. South [7] uses a simplified expression for W for the HE_{11} mode following [15], resulting in approximate expressions for b and some of its derivatives.

Computer programs using the formalisms shown in [2] (modified) and [7] were implemented for comparison purposes. For these comparisons our program accepts as input parameters set b) of Section III.

Table IV shows the values of $\hat{\lambda}$ and V computed by our method and by Marcuse's and South's approaches (E, M, and S, respectively). One observes that Marcuse's results are much closer to ours than are those of South.

Fig. 5 shows curves for material, waveguide, and total dispersion for our method and those presented in [2] and

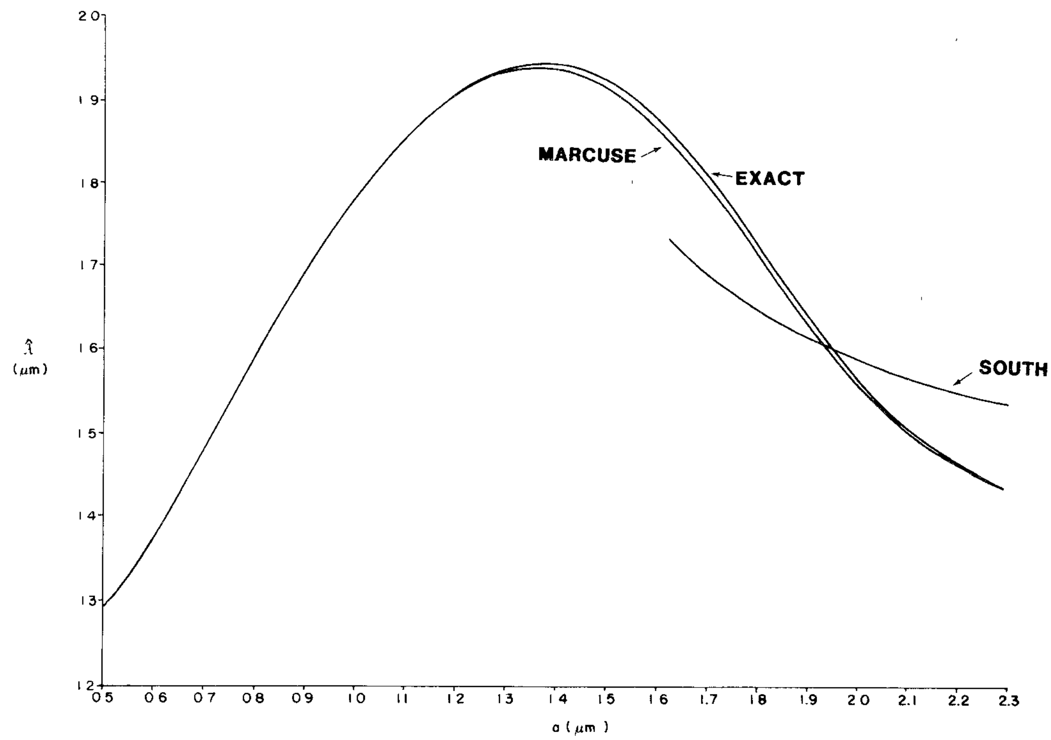

Fig. 6. Variation of the wavelength for minimum total dispersion $\hat{\lambda}$ as a function of core radius a. The core material is 13.5 percent GeO_2–86.5 percent SiO_2 (Sample 3, Table I). The cladding material is 100 percent annealed SiO_2 (Sample 1, Table I).

TABLE V
PERCENTAGE ERRORS FOR $\hat{\lambda}$ FOR DIFFERENT VALUES OF a (FIBER IS THAT OF FIG. 5.) (a) MARCUSE'S METHOD; (b) SOUTH'S METHOD:

CORE RADIUS a(μm)	$\delta\hat{\lambda}$ (μm)
1.10	-0.25
1.40	0.24
1.60	0.66
1.75	0.84
2.10	0.25
2.25	0.06

(a)

CORE RADIUS a(μm)	$\delta\hat{\lambda}$ (μm)
1.65	7.17
1.75	5.81
2.10	-3.99
2.25	-6.61

(b)

[7]. For this case the fiber has a core radius of 1.75 μm and is made with 13.5 percent GeO_2 and 86.5 percent SiO_2 in the core and SiO_2 as the cladding (samples 3 and 1 of Table I, respectively).

Fig. 6 shows the variation of $\hat{\lambda}$ with the core radius for the same fiber materials used in Fig. 5. Table V shows corresponding relative errors in $\hat{\lambda}$ following the procedures of Marcuse a) and of South b). From these results we see that the method proposed in [2] has better characteristics than that used in [7].

Finally, we see that all three approximate methods considered for comparison have shown some difference from ours (mainly the approach of [7]) for the wavelength required for minimum total dispersion. These differences we ascribe mainly to the use of asymptotic expressions for the derivatives of the propagation constant of the dominant mode HE_{11}.

V. CONCLUSIONS

In this paper we presented an analytical and computational approach applicable to the study of pulse dispersion in monomode optical fibers with step refractive index profile. Our approach uses the exact characteristic equation for the computation of the normalized propagation constant b of the fundamental mode in these structures. We also use the exact equation for the total dispersion and numerical techniques for differentiation (for the derivatives of b with respect to V) and interpolation (for computation of $\hat{\lambda}$).

Differences between our approach and some approximate methods analyzed we ascribe to several asymptotic expressions used by those authors. Our method shows excellent results in the asymptotic limit and permits an accurate extension of analysis for a more extensive range of fiber parameters.

Appendix

This appendix shows the equations required to plot the material (D_m) and waveguide (D_g) dispersion curves, shown in Figs. 4 and 5.

The expression for material dispersion is given by [4]

$$D_m = -\frac{\lambda}{c} n''(\lambda) \tag{A1}$$

where $n''(\lambda)$ is given by (20c).

The exact expression for the waveguide dispersion is obtained from the equation for the total dispersion D_T given by (14)–(20), keeping the phase indexes of the core and cladding materials constant with wavelength. Following these steps, we obtain [4]

$$D_g = -\frac{(n_1^2 - n_2^2)V}{2\lambda c n_e}\left\{V\frac{d^2b}{dV^2} + \left(3 - \frac{N_g}{n_e}\right)\frac{db}{dV}\right\} \tag{A2}$$

where

$$N_g = \frac{1}{n_e}\left\{n_2^2 + \left(\frac{V}{2}\frac{db}{dV} + b\right)\theta\right\} \tag{A3}$$

is known as the group index of the waveguide, n_e is given by (7), and

$$\theta = n_1^2 - n_2^2. \tag{A4}$$

Acknowledgment

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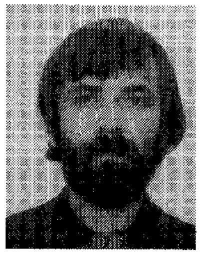

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Equivalent Representations of Nonuniform Transmission Lines Based on the Extended Kuroda's Identity

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Abstract—**Kuroda's identity may be extended to circuits consisting of lumped reactance elements and nonuniform transmission lines. It is shown that these circuits are equivalent to circuits consisting of cascade connections of nonuniform transmission lines whose characteristic impedance distributions are different from original ones, lumped reactance elements, and ideal transformers. If a characteristic impedance distribution $W(x)$ of an original nonuniform transmission line is given, a characteristic impedance distribution $Z(x)$ of a transformed nonuniform transmission line may be uniquely obtained using $W(x)$. Moreover, by using these equivalent transformations, network functions of these transformed nonuniform transmission lines can be derived exactly.**

I. Introduction

It is well known that nonuniform transmission lines show superior transmission responses than the ones of uniform transmission lines. But, it is quite difficult to find the exact network functions of general nonuniform transmission lines from the telegrapher's equation except some nonuniform transmission lines [1]–[12].

On the other hand, we showed that the network functions of a class of nonuniform transmission lines can be exactly derived by using extended Kuroda's identities to mixed lumped and distributed circuits [13].

In this paper, we show a method to extend Kuroda's identities for mixed lumped and nonuniform distributed circuits. Nonuniform transmission lines are shown in the limit of cascaded transmission lines (CTL's) when line length of unit element (UE) approaches zero. By applying Kuroda's identities to circuits consisting of a single stub and CTL's n times, we can show that Kuroda's identities can be extended to circuits consisting of a lumped reactance element and a nonuniform transmission line as the limit case. The transformed circuit becomes the one consisting of a cascade connection of a nonuniform transmission line, a lumped reactance element, and an ideal transformer. Namely, if a characteristic impedance distribution $W(x)$ of an original nonuniform transmission line can be integrated, a characteristic impedance distribution $Z(x)$ of a transformed nonuniform transmission line may be uniquely obtained using $W(x)$. Also, if an exact network function of an original nonuniform transmission line is known, a network function of a transformed nonuniform transmission line can be obtained exactly. We derive exact network functions of several nonuniform transmission lines by applying extended Kuroda's identity to nth order binomial form nonuniform transmission line, exponential transmission line, and hyperbolic secant squared tapered transmission line.

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II. Representation of Nonuniform Transmission Lines

Cascaded transmission lines (CTL's) are shown in Fig. 1(a), where line length and a characteristic impedance of a lossless uniform transmission line (UE) are l/n and W_i $(i=1,2,\cdots,n)$, respectively.

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